A new entropy conservative two-point flux for ideal MHD equations derived from first principles

F. Hindenlang[1] and Gregor Gassner[2]

[1]Department of Numerical Methods in Plasma Physics, Max-Planck Institute for Plasma Physics, Garching, Germany
[2]Department for Mathematics and Computer Science/Center for Data and Simulation Science, University of Cologne, Cologne, Germany

It was shown by Gassner [1] that the Discontinuous Spectral Element Method with Gauss-Lobatto collocation points (DGSEM-GL) belongs to the class of diagonal-norm SBP-SAT schemes, which are the basis of provably stable discretizations for linear and nonlinear equation systems. The SBP property was first constructed for Finite Difference methods, and all SBP schemes can be written as a modified high order Finite Volume scheme [2].

For a stable DGSEM-GL, the standard volume integral is replaced by a special difference using a two-point flux, while keeping the same dimension-by-dimension operator structure and conservation properties. If the same two-point flux is used as a numerical flux, the resulting scheme has for example discrete kinetic energy preservation (KEP) [3] or entropy conservation (EC)[4]. In addition, it allows to easily discretize quadratic and cubic product rules arising in split-form equations [5].

The definition of a two-point flux for KEP or EC is not unique, and many different choices have been proposed for Euler and ideal MHD equations. In this talk, we explain in detail the SBP property and how the two-point flux formulation connects the DGSEM-GL with Finite Volume schemes. We present a rigorous derivation of a new EC flux for the ideal MHD equations, starting from first principles of the ideal MHD equations. This enables us to analyze a-priori the discrete behavior of kinetic and magnetic energy, pressure, the Lorentz force and the magnetic divergence. Interestingly, we found that for a cartesian mesh, the dissipation-free DGSEM-GL with the new EC flux will preserve zero divergence of the magnetic field discretely. We will also demonstrate these properties for three-dimensional ideal MHD testcases, computed with the Fluxo framework (`github.com/project-fluxo`).

References

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