A L2-norm regularized incremental-stencil WENO scheme for compressible flows

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Abstract

In the simulation of compressible flow with a broadband of length scales and discontinuities, using the scheme with incremental width is promising. Since in the region with discontinuities, low-order candidate stencils are switched on, high-order stencils will contribute more in the non-discontinuity region with small scale fluctuations, which makes the numerical scheme more robust with less numerical dissipation. Therefore, high order schemes with incremental width stencil are developed for simulation of compressible flows with complex flow structures [1, 2]. However, in smooth region, large weights may be imposed on low-order stencils due to the lack of high order derivatives of smoothness indicator compared with that of high-order stencils, which would degrade the order accuracy especially in the region near a critical point. In this paper, same with the stencil selection of Ref. [2], a L2-norm regularization based [3] incremental-stencil WENO scheme is proposed. In order to cope with the discrepancy of the smoothness indicator between high-order and low-order stencils, L2-norm regularization is introduced into the evaluation of smoothness of the stencils by adding a regularized term into the smoothness indicators. A number of testing cases suggest that the present scheme with the modification is more robust and has better wave-resolution capabilities than other schemes with the same full stencil. As shown in Fig. 1, the proposed WENOIS-L2 scheme preform better than WENO-JS and WENO-Z scheme. The small-scale structures at high-frequency are well solved and the fluctuation amplitude is maintained up to 4.61 at $x = 6.85$ which is even higher than that of WENO-CU6(see Fig.5 of [4]). Fig. 2 gives the density and pressure profiles obtained by the proposed scheme without using any positive preserving method. We can observe that the results are in good agreement with those in Zhang and Shu [5] (their Fig.4.6) with more scales and flow structures resolved.

Keywords: WENO scheme, incremental stencil, L2-norm regularization, discontinuity detector, compressible flow

References

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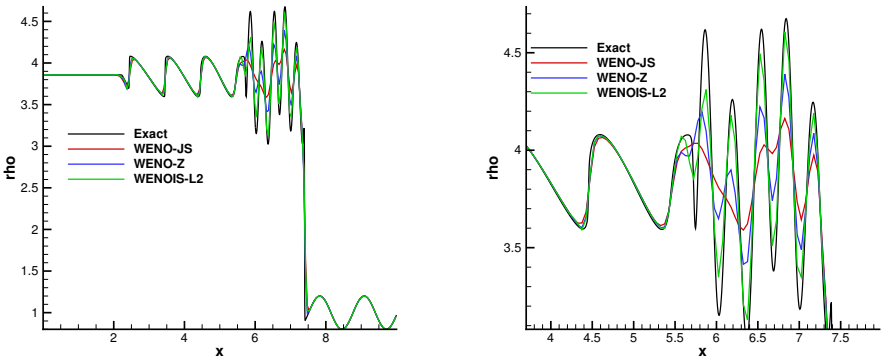

(a) (b)

Figure 1: Shu-Osher problems: solution from WENO-JS, WENO-Z and WENOIS-L2 schemes on 200 uniform grid points. WENOIS-L2 denotes the proposed L2-norm regularized incremental-stencil scheme. (a) The comparison of density profile; (b) the enlarge portion of (a).

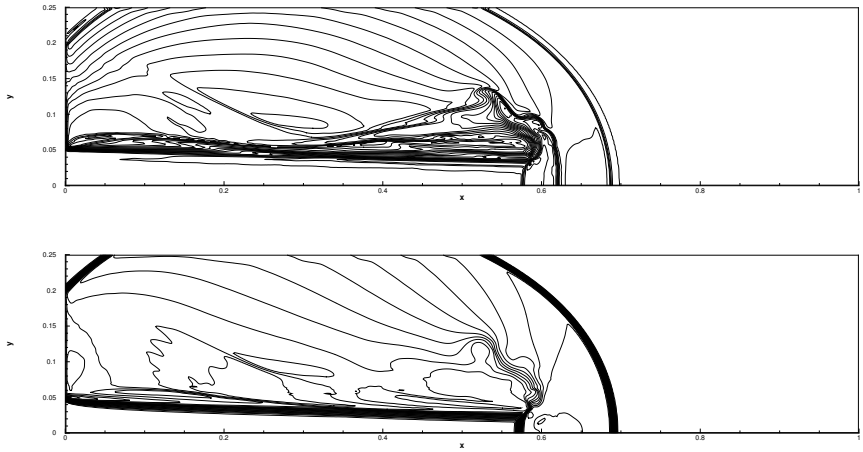

Figure 2: Mach-2000 jet problem: (upper) 30 density contours of logarithmic scale from -4 to 4; (lower) 30 pressure contours of logarithmic scale from -1 to 13.

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