Towards a generalised limiter for nonlinear conservation laws through domain adaptation

Rémi Abgrall, Maria Han Veiga

February 2019

1 Abstract

In this work, we are interested in constructing a parameter free limiter for hyperbolic conservation laws, which is agnostic to the mathematical model and underlying numerical method. Based on work such as [1, 2], a neural network is trained to identify cells which are in need of limiting without having to fix a parameter which often depends on initial data, which is a common practice for many limiters available in literature[3, 4]. When such parameter is poorly tuned, the cost is shown through excessive smooth extrema clipping, excessive dissipation or unstable schemes.

The work in [2] is extended for 2-d problems defined on a cartesian mesh for Runge-Kutta discontinuous Galerkin scheme. To explore the idea of domain adaptation, we test the performance of this shock detector on a residual distribution scheme (cartesian mesh and triangular mesh), when performing feature projection and when retraining the neural network with a reduced dataset generated with the residual distribution scheme.

In particular, the generalisation of a limiter can be seen as a transfer learning problem, where the *source* task is shock detection in a RKDG scheme, and the *target* task, a shock detection with a different numerical scheme/mesh. In this work, we perform an assymetric feature transformation [5], to align the target feature space to the source feature space. We compare the performance of simply performing feature transformation and retraining using a smaller target dataset. We compare the performance of this *transferred* limiter with different stabilisation methods for residual distribution.

References

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