High order deferred correction residual distribution schemes for stiff relaxation problems with implicit treatment

Davide Torlo* and Rémi Abgrall†

University of Zürich

Many physical model largely used in numerical analysis are the macroscopic limit of relaxation models, for example Euler and Navier-Stokes are limits of Boltzmann equation. Many other systems present a relaxation given by some physical parameters, for instance viscoelasticity problems, multiphase flows. Others contain an artificial relaxation term to add viscosity or to make the hyperbolic part linear, hiding the nonlinearities in the source term. The scope of our schemes is to solve a class of hyperbolic relaxation problems, which contain stiff source terms. Moreover, we want to obtain a high order accurate in space and time scheme, using the residual distribution formulation for the space discretization and the deferred correction method to obtain a high order time integration. [1]

To deal with the stiff relaxation term, we have to recur to an implicit treatment of this source term, indeed, with classical explicit methods, it is necessary to use discretization scales of the size of the relaxation parameter, which is often unfeasible. The IMEX (implicit explicit) discretization we propose to use is straightforward and treats the stiff source implicitly and the other terms explicitly. The accuracy order in time obtained with this procedure is first order. [7]

The high order accuracy in time is given by the Deferred Correction (DeC) method [2, 6, 3], which combines two schemes (an explicit or IMEX first order one and a high order totally implicit one) in an iterative way. This allows in few steps to reach the accuracy of the second scheme without the need to solve it implicitly. This algorithm is quite general and it does not need special care for different order of accuracy, in contrast with IMEX Runge–Kutta (RK) methods, where each order corresponds to a different set of weights and there are different property to be verified for the RK scheme to be stable.

For space discretization we use Residual Distribution schemes (RD) [2, 5], that are a class of schemes which can be shaped into different well known finite volume, finite differences or finite element schemes. They are very versatile, with compact stencil and require only a loop through the elements of the mesh and one through the degrees of freedom to be computed.

The final scheme obtained is proven to be asymptotic preserving, i.e., it is able to automatically switch from the microscopic to the macroscopic regime accordingly to the relaxation parameters. We tested it with various test cases, in particular with a kinetic model proposed by [4], where any hyperbolic system can be recast into a bigger relaxation

*davide.torlo@math.uzh.ch

†remi.abgrall@math.uzh.ch

problem with some extra diffusion given by the relaxation terms. Other models are tested and presented, both in 1D and 2D for different order of accuracy, showing the quality of the scheme.

Acknowledgments: This research is funded by ITN ModCompShock project in the European Union's Horizon 2020 research and innovation program under the Marie Sklodowska-Curie grant agreement No 642768.

References

[1] Torlo, D. and Abgrall, R., *Asymptotic Preserving Deferred Correction scheme*, submitted, December 2019.

[2] R. Abgrall. *High order schemes for hyperbolic problems using globally continuous approximation and avoiding mass matrices.* Journal of Scientific Computing, January 2017.

[3] A. Dutt, L. Greengard and V. Rokhlin. *Spectral Deferred Correction Methods for Ordinary Differential Equations.* BIT Numerical Mathematics. 40 (2) (2000): 241-266.

[4] Aregba-Driollet, D. and Natalini, R., *Discrete Kinetic Schemes for Systems of Conservation Laws.* SIAM J. Numerical Analysis 37 (2000): 1973–2004.

[5] M. Ricchiuto and R. Abgrall. *Explicit Runge-Kutta residual distribution schemes for time dependent problems: Second order case.* J. Comput. Phys. 229 (16) (2010): 5653 - 5691.

[6] M. Minion. *Semi-implicit spectral deferred correction methods for ordinary differential equations.* Communication in Mathematical Physics 1 (3) (2003): 471-500.

[7] S. Boscarino, L. Pareschi and G. Russo, *Implicit-Explicit Runge–Kutta Schemes for Hyperbolic Systems and Kinetic Equations in the Diffusion Limit.* SIAM Journal on Scientific Computing. (2011).