High-order invariant domain preserving ALE approximation of hyperbolic systems *

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Abstract

The objective of the present paper to propose a second-order continuous finite element technique for solving hyperbolic systems in the arbitrary Lagrangian Eulerian framework (ALE). This is done by revisiting the results from [3] and adapting to the ALE setting the recently proposed convex limiting technique from [1, 2]. The main property of the method presented in the paper is that, provided the user-defined ALE velocity is reasonable, the approximate solution produced by the algorithm is formally second-order accurate in space, is conservative, satisfies the so-called discrete geometric conservation law, and preserves as many convex invariant sets of the hyperbolic system as desired by the user. The time stepping is explicit, the approximation in space is done with continuous finite elements. The second-order accuracy is numerically shown to hold in the maximum norm for smooth solutions of nonlinear scalar conservation equations and with smooth solutions of the compressible Euler equations.

Key words

Conservation equations, hyperbolic systems, Arbitrary Lagrangian Eulerian, moving meshes, invariant domains, high-order method, convex limiting, finite element method.

AMS subject classifications. 65M60, 65M10, 65M15, 35L65

References

[1] J.-L. Guermond, , M. Nazarov, B. Popov, and I. Tomas. Second-order invariant domain preserving approximation of the euler equations using convex limiting. *SIAM, J. Sci. Comput.*, 2017. Submitted.

[2] J.-L. Guermond, , B. Popov, and I. Tomas. Invariant domain preserving discretization-independent schemes and convex limiting for hyperbolic systems. *Comput. Methods Appl. Mech. Engrg.*, 2018. Submitted.

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[3] J.-L. Guermond, B. Popov, L. Saavedra, and Y. Yang. Invariant domains preserving arbitrary Lagrangian Eulerian approximation of hyperbolic systems with continuous finite elements. *SIAM J. Sci. Comput.*, 39(2):A385–A414, 2017.