On Strong Stability of Explicit Runge-Kutta Methods for Nonlinear Problems

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Explicit Runge-Kutta methods are classical and widespread techniques in the numerical solution of ordinary differential equations (ODEs). Considering partial differential equations, spatial semidiscretisations can be used to obtain systems of ODEs that are solved subsequently, resulting in fully discrete schemes. However, certain stability investigations of high-order methods for hyperbolic conservation laws are often conducted only for the semidiscrete versions.

Here, strong stability (also known as monotonicity) of explicit Runge-Kutta methods for ODEs with nonlinear and semibounded (also known as dissipative) operators is investigated. Contrary to the linear case studied in [2–4], it is proven that many strong stability preserving (SSP) schemes of order two or greater are not strongly stable for general smooth and semibounded nonlinear operators. Additionally, it is shown that there are first order accurate explicit SSP Runge-Kutta methods that are strongly stable (monotone) for semibounded (dissipative) and Lipschitz continuous operators.

This talk is based on [1].

References

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