High-Order Methods on Summation by Parts Form for the Magnetic Induction Equation

Hendrik Ranocha, Katharina Ostaszewski, Philip Heinisch

April 2019

The magnetic induction equation, possibly enhanced with nonlinear Hall term, can be used to model magnetic fields, e.g. in astrophysical plasma environments. In order to give reliable results, numerical simulations should be carried out using effective and efficient schemes. Thus, high-order stable schemes are investigated here.

Following the approach provided recently in [1, 2], an energy analysis for both the linear and the nonlinear induction equation including boundary conditions is performed at first. Novel outflow boundary conditions for the Hall induction equation are proposed, resulting in an energy estimate. Based on an energy analysis of the initial boundary value problem at the continuous level, semidiscretisations using summation by parts (SBP) operators and simultaneous approximation terms are created. Mimicking estimates at the continuous level, several energy stable schemes are obtained in this way and compared in numerical experiments. Moreover, stabilisation techniques correcting errors in the numerical divergence of the magnetic field via projection methods are studied from an energetic point of view in the SBP framework. The resulting numerical methods are implemented in OpenCL and have been published as open source code [3].

This talk is based on [4].

References

[1] J. Nordström. 'A Roadmap to Well Posed and Stable Problems in Computational Physics'. In: *Journal of Scientific Computing* 71.1 (2017), pp. 365–385. DOI: 10.1007/s10915-016-0303-9.

[2] J. Nordström and C. La Cognata. 'Energy stable boundary conditions for the nonlinear incompressible Navier-Stokes equations'. In: *Mathematics of Computation* (2018). DOI: 10.1090/mcom/3375.

[3] H. Ranocha, K. Ostaszewski and P. Heinisch. *InductionEq. A set of tools for numerically solving the nonlinear magnetic induction equation with Hall effect in OpenCL.* https://github.com/MuMPlaCL/InductionEq. Sept. 2018. DOI: 10.5281/zenodo.1434409.

[4] H. Ranocha, K. Ostaszewski and P. Heinisch. *Numerical Methods for the Magnetic Induction Equation with Hall Effect and Projections onto Divergence-Free Vector Fields*. Submitted. Oct. 2018. arXiv: 1810.01397 [math.NA].