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ENATE, a high-order scheme for cartesian grids with arbitrary expansion/contraction ratios

Antonio Pascau[a,*], Víctor J. Llorente[a]
[a]Dept. Sci. & Tech. of Materials and Fluids, University of Zaragoza
María de Luna, 50018, Zaragoza
[*]Corresponding author: pascau@unizar.es

ENATE is a high-order scheme that provides the exact solution of the one-dimensional transport equation

$$\frac{\partial}{\partial x}\left(\rho u\phi - \Gamma\frac{\partial \phi}{\partial x}\right) = S(x)$$

for arbitrary coefficients and source. The algebraic equation that links the values of ϕ at three points contains as coefficients several integrals whose integrands are dependent on the actual spatial variation of the convective, diffusive and source factors. The sizes of adjacent subintervals are totally arbitrary. We can use a small subinterval, for instance, followed by one whose size is ten times as great and followed by one of the previous size.

In 2D the transport equation is

$$\nabla \cdot \mathbf{F} = S(x,y)$$

being $\mathbf{F}$ the total flux vector

$$\mathbf{F} = \left(\rho u\phi - \Gamma\frac{\partial \phi}{\partial x}\right)\mathbf{e}_x + \left(\rho v\phi - \Gamma\frac{\partial \phi}{\partial y}\right)\mathbf{e}_y = F_x\mathbf{e}_x + F_y\mathbf{e}_y$$

In 2D we split the equation in two

$$\frac{\partial}{\partial x}\left(\rho u\phi - \Gamma\frac{\partial \phi}{\partial x}\right) = \frac{1}{2}S(x,y) + \beta(x,y)$$
$$\frac{\partial}{\partial y}\left(\rho v\phi - \Gamma\frac{\partial \phi}{\partial y}\right) = \frac{1}{2}S(x,y) - \beta(x,y)$$

where β is an unknown scalar field. The target value of β is one such that the two equations provide the same value for ϕ. To calculate the β-field, the previous equation is derived with respect to β

$$\frac{\partial}{\partial x}\left(\rho u\left(\frac{\partial \phi}{\partial \beta}\right) - \Gamma\frac{\partial}{\partial x}\left(\frac{\partial \phi}{\partial \beta}\right)\right) = 1$$
$$\frac{\partial}{\partial y}\left(\rho v\left(\frac{\partial \phi}{\partial \beta}\right) - \Gamma\frac{\partial}{\partial y}\left(\frac{\partial \phi}{\partial \beta}\right)\right) = -1$$

Once the $\partial\phi/\partial\beta$-field has been obtained β is updated following the iterative procedure given below

$$\beta^{new} = \beta^{old} + \frac{(\phi_{target} - \phi_{current})}{\partial\phi/\partial\beta} \quad ; \quad \phi_{target} = 1/2(\phi_1 + \phi_2)$$

ϕ_1 and ϕ_2 are the values of ϕ, solution of the two equations. This procedure has been applied to a model of ocean circulation put forward by Stommel who derived a transport equation for the stream function. We will compare our results with others found in the literature.