Compact Approximate Taylor methods for systems of conservation laws..

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This paper focuses on the extension of Lax-Wendroff methods to nonlinear systems of conservation laws. The main difficulty comes from the transformation of time derivatives into spatial derivatives using the equations. A first strategy to do this is given by the Cauchy-Kovalevskaya (CK) procedure: see [1]. The main drawback of this strategy comes from the fact that the CK procedure leads to expressions whose number of terms grow exponentially what implies high computational costs and difficult implementations. In [2] an alternative has been presented based on an Approximate Taylor (AT) method in which the time derivatives are approximated using high-order centered differentiation formulas combined with Taylor approximations in time that are computed in a recursive way. Nevertheless AT schemes are not proper generalizations of Lax-Wendroff methods: they have $(4p+1)$-point stencils and worse linear stability properties than the original Lax-Wendroff methods that have $(2p+1)$-point stencil. Nevertheless, they can be stabilized by using one WENO reconstruction per spatial cell and time step: the resulting methods give good results under a CFL-0.5 condition typically.

In order to design numerical methods that are proper generalization of Lax-Wendroff methods, a compact variant of the AT procedure is introduced here: first, the conservative expression of the high- order Lax-Wendroff methods is considered; then, the derivatives appearing in the expression of the numerical flux are computed using $2p$-point differentiation formulas. This strategy lead to Compact Approximate Taylor (CAT) methods that have $(2p+1)$-point stencil and order of accuracy $2p$. They reduce to the Lax-Wendroff method when applied to a linear systems and thus they are linearly L^2- stable under a CFL-1 condition. As it happens with its linear counterpart, CAT methods lead to spurious oscillations near discontinuities. In order to cure them, two shock-capturing techniques are considered here: a flux-limiter technique and the use of a WENO reconstruction per cell and time step. A number of test cases will be shown: the linear transport equation, Burgers equation, and the 1D compressible Euler system are considered.

References

[1] J.Qiu, C.-W. Shu. *Finite difference WENO schemes with Lax-Wendroff-Type time discretizations.* SIAM J. Sci. Comp. 24 (2003), 2185–2198.

[2] D. Zorio, A. Baeza, P. Mulet. *An Approximate Lax-Wendroff-Type Procedure for High Order Accurate schemes for Hyperbolic Conservation Laws.* J. Sci Comput 71(2017), 246–273.