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ENATE, a high-order scheme for convection-diffusion problems

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We present new applications of a novel Finite-Volume (FV) high-order exponential scheme named ENATE (**E**nhanced **N**umerical **A**pproximation of a **T**ransport **E**quation) for solving the steady elliptic equation,

$$\nabla \cdot \mathbf{F} = S$$

being the total flux

$$\mathbf{F} = \left(\rho u\phi - \Gamma\frac{\partial\phi}{\partial x}\right)\mathbf{e}_x + \left(\rho v\phi - \Gamma\frac{\partial\phi}{\partial y}\right)\mathbf{e}_y = F_x\mathbf{e}_x + F_y\mathbf{e}_y$$

Our main variables are ϕ as the transport variable; ρu and ρv, the convective terms that meet continuity; Γ, the diffusion parameter; S, sources. ENATE supports the exact solution of a nonhomogeneous with variable coefficients convection-diffusion equation in one-dimension. When the problem is multidimensional, a splitting operator method is used in this way: we solve with ENATE each transport equation in both coordinates by setting the derivatives of the flux in the other directions as new source terms. By doing so, the discrete equation writes

$$\mathcal{A}_C\phi_C - \sum_{i\in\mathcal{N}}\mathcal{A}_i\phi_i = b_C - (b_C^{\partial F})^{old}$$

We use a FV notation where the surrounding nodes, $\mathcal{N}$, of the Central node are the West, East, North and South. The influence coefficients, $\mathcal{A}$, are based in some integrals of convection and diffusion. We can prove that the $\mathcal{A}$'s are always positive and in some special cases, $\mathcal{A}_C \geq \sum\mathcal{A}_i$. Then, in those cases the matrix of ENATE is a M-matrix.

On the other hand, the discrete source, b_C, is based in some integrals of S and $b_C^{\partial F}$ similar to the previous one but setting $\partial F_x/\partial x$ and $\partial F_y/\partial y$ as S. This new discrete source must be evaluated accurately in an old iteration. Start off by solving the discrete system without $b_C^{\partial F}$, and then keep updating $b_C^{\partial F}$ with those ϕ's. With some numerical tests, we will assess the high-order scheme.