Approximate Riemann Solution of the Generalized Riemann Problem for Advection Diffusion Equations

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Abstract

In the numerical solution of nonlinear advection diffusion equations, the conventional approach for the flux calculation is to calculate the hyperbolic and parabolic part separate from each other. In this approach, the hyperbolic flux introduces numerical dissipation without taking into account the physical one. This may falsify the physical dissipation introduced by the corresponding diffusion flux.

In our work, we concentrate ourselves on the solution of the generalized Riemann problem. For the hyperbolic case, an approximate solution was proposed by Ben-Artzi and Falcovitz [1]. In the parabolic case, an exact solution for linear equations has been used by Gassner et al. [2]. In this talk we show the construction of an approximate solution for advection diffusion equations. It is based on the weak formulation of the Riemann problem and is solved within an explicit space-time discontinuous Galerkin approach with two space-time elements. The Riemann solver is then used to define the numerical flux for a finite volume method.

Numerical results for the viscous Burgers equation clearly indicate a reduction of numerical dissipation compared to the conventional approach. We will discuss the challenges undertaken to extend the approach to systems of equations and will conclude with some preliminary results of its application to the Euler equations with heat conduction.

References

[1] M. Ben-Artzi nd J. Falcovitz, *A Second-Order Godunov-Type Scheme for Compressible Fluid Dynamics*, Journal of Computational Physics, Vol. 55, pp. 1-32, 1984.

[2] G. Gassner, F. Lörcher and C.-D. Munz, *A Contribution to the Construction of Diffusion Fluxes for Finite Volume and Discontinuous Galerkin Schemes*, Journal of Computational Physics, Vol. 224, pp. 1049-1063, 2007.