High order path-conservative ADER discontinuous Galerkin schemes for the GRMHD equations

Francesco Fambri[†], Michael Dumbser[†], Olindo Zanotti[†], Luciano Rezzolla[‡§], Sven Köppel[‡§]

francesco.fambri@unitn.it

† *Laboratory of Applied Mathematics, University of Trento, Trento, Italy*
‡ *Institute for Theoretical Physics, Frankfurt, Germany*
§ *Frankfurt Institute for Advanced Studies, Frankfurt, Germany*

In this talk we present a new class of high-order accurate numerical algorithms for solving the equations of general-relativistic ideal magnetohydrodynamics (GRMHD) in curved spacetimes.

The GRMHD governing partial differential equations are solved via a new family of fully-discrete and arbitrary high-order accurate path-conservative discontinuous Galerkin (DG) finite-element methods combined with adaptive mesh refinement (AMR) and time accurate local timestepping (LTS). An important and novel aspect of our approach is the interpretation of the source terms in the GRMHD equations that account for the gravitational field in curved spacetimes as nonconservative products, instead of the usually employed algebraic source terms. In this work we assume the background spacetime to be given and static, i.e., we make use of the Cowling approximation.

In order to deal with shock waves and other discontinuities or singularities, the high-order DG schemes are supplemented with a novel a-posteriori subcell finite-volume limiter, which makes the new algorithms as robust as classical finite-volume methods at shocks and discontinuities, but also as accurate as unlimited high-order DG schemes in smooth regions of the flow.

We validate the correctness of the formulation and the corresponding discretization through a series of numerical tests in one, two and three spatial dimensions.

In this talk, a novel approach for the treatment of general equations of states will be also briefly discussed. In particular, this new numerical algorithm allows to represent general tabulated EOS at the aid of piecewise polynomials in phase space in order to produce an approximation to the relation $p = p(\rho, e)$, where p is the fluid pressure, ρ its density and e the specific internal energy. The new approach for general EOS has been validated with available reference solutions for the ideal gas, but represented in the general form $p = p(\rho, e)$.

References

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