A high order discontinuous Galerkin scheme for a hyperbolic relaxation system for dispersive non-hydrostatic water waves

C. Escalante*, M. Dumbser†, M.J. Castro ‡

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Abstract

In this talk we present a novel set of first-order hyperbolic equations that can model dispersive non-hydrostatic free surface flows. The governing PDE system is obtained by making a hyperbolic approximation of the non-hydrostatic free-surface flow model recently derived by Sainte-Marie *et al.* in [1], which describes the propagation of dispersive waves in shallow waters. Our new hyperbolic approximation is based on an augmented system in which the divergence constraint of the velocity is coupled with the other conservation laws via an evolution equation for the depth-averaged non-hydrostatic pressure, similar to the hyperbolic divergence cleaning applied in generalized Lagrangian multiplier methods (GLM) for magnetohydrodynamics (MHD). We suggest a formulation in which the divergence errors of the velocity field are transported with a large but finite wave speed that is directly related to the maximal eigenvalue of the governing PDE. .

We then use arbitrary high order accurate (ADER) discontinuous Galerkin (DG) finite element schemes see ([2, 3]) with an *a posteriori* subcell finite volume limiter via the MOOD approach, see [4, 5, 6, 7]. The final scheme is highly accurate in smooth regions of the flow and very robust and positive preserving for emerging topographies and wet-dry fronts. It is well-balanced making use of a path-conservative formulation of HLL-type Riemann solvers based on the straight line segment path. The resulting subcell finite volume limiter used in this paper is the natural extension of the numerical scheme presented in [8]. Furthermore, the proposed ADER-DG scheme with *a posteriori* subcell finite volume limiter adapts very well to modern GPU architectures, resulting in a very accurate, robust and computationally efficient computational method for non-hydrostatic free surface flows. The new model proposed here has been applied to idealized academic benchmarks such as the propagation of solitary waves, as well as to more challenging physical situations that involve wave runup on a shore including wave breaking in both one and two space dimensions. In all cases the achieved agreement with analytical solutions or experimental data is very good, thus showing the validity of both, the proposed mathematical model and the numerical solution algorithm.

Keyworkds: non-hydrostatic shallow water flows, hyperbolic reformulation, ADER discontinuous Galerkin schemes, path-conservative finite volume methods, breaking waves, efficient parallel implementation on GPU.

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*Dpto. Matemática Aplicada. Universidad de Málaga, Spain

†Laboratory of Applied Mathematics, Department of Civil, Enviromental and Mechanical Engineering, University of Trento, Italy

‡Dpto. AM,EIO y Matemática Aplicada. Universidad de Málaga, Spain

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