An order-adaptive compact approximation Taylor method for systems of conservation laws.

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We present an order-adaptive finite difference numerical method for systems of conservation laws. The method, called Adaptive Compact Approximation Taylor method (ACAT), uses centered $(2p+1)$-point stencils, where p may take values in $\{1, 2, \ldots, p_{max}\}$ according to a family of smooth indicators in the stencils. The method writes as a combination between a robust first order scheme and $2p$-order generalized Lax-Wendroff methods, so that it is first order near shocks and of order $2p$ in smooth regions, where $(2p+1)$ is the size of the biggest stencil in which large gradients are not detected. A new class of smooth indicators and the stability analysis (based on [1]) will be presented.

For nonlinear problems, the original LW procedure requires the conversion of the time derivatives to spatial derivatives through the so-called Cauchy-Kovalevskaya process, what may increase dramatically the computational cost (see [2]). To avoid this, we adapt the Compact Approximate Taylor Method (CAT) introduced in [3], by including an adaptive formulation in the numerical differentiation formulas plus a new class of smooth indicators. In comparison with the Approximate Taylor methods presented in [4], WENO flux reconstruction are not needed and values of the CFL parameter close to 1 can be used, what reduces significantly the computational cost. The general structure of the method and a number of numerical tests will be shown, in which the results are compared with those provided by standard WENO methods (see [5]) and the Approximate Taylor methods introduced in [3] and [4].

References

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