High order semi-implicit discontinuous Galerkin methods for natural convection problems

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The purpose of this work is to introduce a new high order semi-implicit discontinuous Galerkin method (DG) for natural convection problems with small temperature disturbances in two and three space dimensions. The model to be solved involves the incompressible Navier-Stokes equations coupled with an energy equation for the temperature. Throughout this talk we will make use of the Boussinesq assumption.

The efficient DG scheme introduced in [1, 2, 3] is extended to solve the additional energy equation together with the gravity source terms in the momentum equation due to buoyancy effects. The computational domain is discretized using a staggered unstructured mesh of the face-type. The velocity is computed on the dual grid whereas pressure and temperature are obtained on the primal simplex mesh. In order to assess the developed methodology several classical test cases are considered showing a good agreement with available numerical reference data. As an alternative to an explicit upwind scheme for the discretization of the nonlinear convective terms, a semi-Lagrangian approach has been developed. This approach allows the use of much larger time steps so that the computational cost decreases substantially. The main strengths and drawbacks for both methodologies are analysed by using several benchmarks.

References

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