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A conservative limiting method for bicompact and finite-element schemes

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Development of high order numerical methods for partial differential equations, including hyperbolic equations, remains an important area of modern numerical mathematics. Bicompact schemes belong to this class of methods. In addition, bicompact schemes combine several positive properties: even high order (4th, 6th, and so on) of spatial approximation on a stencil that occupies only one mesh cell; an ability to choose time stepping method; effective implementation and, at the same time, implicit approximation in all directions; good spectral properties [1].

As all high order schemes, bicompact ones require some kind of limiting in presence of strong discontinuities. Previously, it has been done using the hybrid scheme approach [2–5]. As the latter possesses some advantages, it also has a significant drawback: bicompact schemes cease to be conservative if they are monotonized through the hybrid scheme, which is not desirable.

The aim of this work is to eliminate the aforementioned problem. It is shown, that the high order bicompact approximation of spatial derivatives can be described in terms of a finite element approach. Moreover, an analogy between bicompact and Galerkin type schemes is established. A new conservative limiting method for bicompact schemes is constructed on the basis of their finite element treatment. The main idea of the proposed method is application of hybrid scheme weighting factors to slope limiting. The designed method may be utilized by other finite element schemes. Finally, bicompact schemes with conservative limiting are verified on a number of one- and multidimensional gas dynamics problems. Numerical results demonstrate good quality of solutions computed by these schemes. An example of our results in case of the Shu-Osher problem is given of Figs. 1–2.

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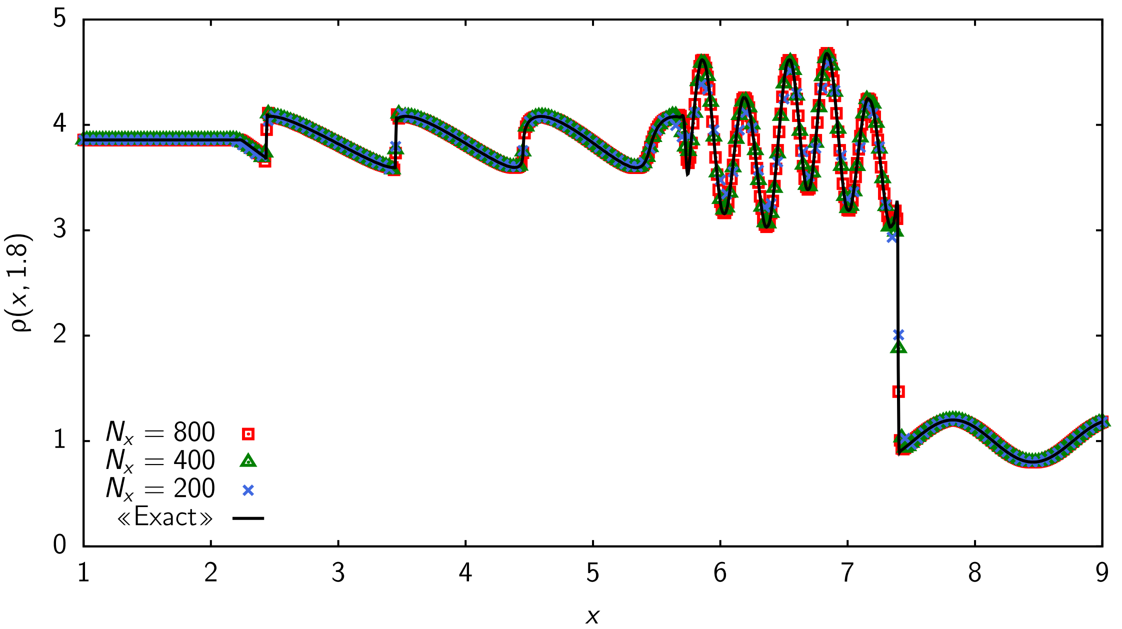

Fig. 1. Density profiles in the Shu-Osher problem at $t = 1.8$, computed by the bicompact scheme of 3rd order in time and 4th order in space, CFL number $= 0.5$

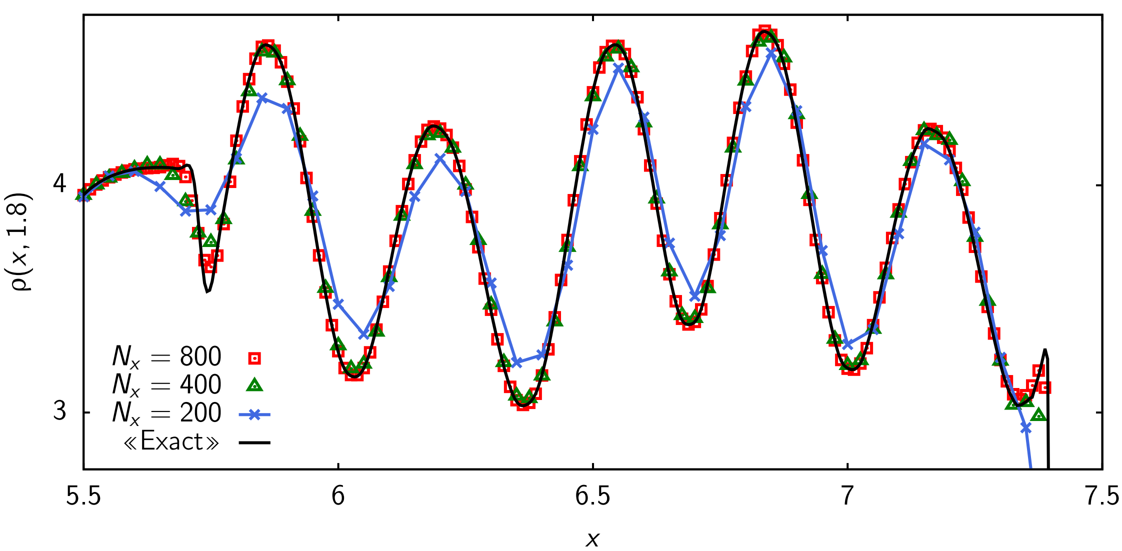

Fig. 2. Close-up of the Fig. 1 at $x \in [5.5, 7.5]$, $\rho \in [2.75, 4.75]$