The need for structure preserving methods in continuum physics

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This talk aims in motivating the developers of numerical methods for continuum mechanics models to develop *structure preserving methods*. Using a sufficiently large class of Symmetric Hyperbolic Thermodynamically Compatible (SHTC) equations [3] as an example, we discuss the necessity of preservation of *structural properties* of PDE systems at the discrete level in order to improve physical consistency of the numerical solution. The SHTC class of equations provides a first-order hyperbolic framework for many branches of continuum mechanics including fluid and solid mechanics [2, 1], transfer phenomena (equilibrium and non-equilibrium momentum, heat, mass and electric charge transfer), multi-phase flows and solid-fluid mixtures (poroelasticity), relativistic dissipative continuum mechanics, dispersive wave propagation phenomena, etc. The SHTC equations, therefore, can be viewed as a convinient modeling tool for those who is working with and developing numerical methods for hyperbolic PDEs. Nevertheless, obtaining a physically consistent numerical solution to an SHTC system is not a trivial task due to the presence of non-conservative products and stiff relaxation source terms. So far, we relied on the family of ADER-DG and ADER-FV schemes [2] which can effectively address both these issues and allows obtaining a very accurate solution. However, being a family of methods for general hyperbolic equations, the ADER family cannot preserve some important structural properties of the SHTC equations. Within such structural features of PDEs, we shall consider

1. **Overdetermination** of a PDE system (i.e. when the number of equations is larger than the number of unknowns),
2. **Involution constraints** (i.e. when the physical solution has to respect some additional stationary constraints),
3. **Hamiltonian structure** (i.e. when the PDE system can be generated by the corresponding Poisson brackets).

All three features are inherent in the SHTC class of equations and interrelated.

In this talk, we shall discuss the structural properties of the SHTC equations using the examples from fluid and solid dynamics, magnetohydrodynamics, turbulence and dislocation modeling, surface tension modeling, dispersive wave propagation, etc. We shall also discuss several routes to build a structure preserving scheme for the SHTC class of equations.

References

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