# Kinetic energy preserving split form flux reconstruction for the compressible Euler equations at Gauss nodes

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High order methods have been shown to provide more accurate solutions for the same degrees of freedom. Flux reconstruction (FR) [1] methods represent an arbitrarily high-order family of methods that employ unstructured body-fitted grids to enable simulations over complex geometries. However, high order methods are particularly susceptible to instabilities, often due to aliasing, which is caused by non-linear terms in the governing equations. One technique to alleviate this, initially developed in the high order finite difference community [2], is using split forms. Split forms were originally developed as skew-symmetric formulations for energy preservation. In recent years, they have been modified to introduce properties into the discretized system, such as kinetic energy preservation and entropy preservation.

With the flux differencing form [3], split forms developed in the finite difference and finite volume community can be easily extended to FR with Gauss Lobatto points. However, extension to Gauss points is not straightforward due to the fact that they do not include boundary nodes. It has been shown that using Gauss points can lead to more accurate solutions than Gauss Lobatto points due to the higher accuracy of the associated quadrature rule. Using split forms that preserve kinetic energy and entropy can be shown to be more robust for simulating high Reynolds' number flows. But, if split forms are extended to Gauss points, they are not conservative anymore. In the present work, first we show that using the generalized summation-by-parts (SBP) property of FR at Gauss points, any quadratic or cubic term can be made conservative. Next, we use the use the Kennedy and Gruber (KG)

[4] split form, modify it to be conservative, and then introduce an additional variable to make it kinetic energy conservative. Using this new formulation, we show improved accuracy against the KG split form with Gauss Lobatto points. In addition, we show that new formulation improves robustness over the un-split formulation for Gauss points. Finally we show the kinetic energy dissipation rate comparison for the Taylor Green Vortex flow.

## References


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