New Multigrid Preconditioners for DG Methods

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Abstract

We consider compressible turbulent flow problems which arise for example in the design of next generation jet engines, air frames, wind turbines, or star formation.

In this talk we study the discontinuous Galerkin spectral element method with Gauss-Lobatto-Legendre nodes (DGSEM-GL) for nonlinear conservation laws. It was proven that DGSEM-GL can be written as a specific finite volume (FV) method ([2],[3]). This allows to apply known theory from FV methods to the DGSEM-GL formulation.

The nonlinear system arising from an implicit DGSEM discretization is solved with a preconditioned Newton-Krylov method. Constructing a good preconditioner allows to set up a fast and efficient DG solver. Multigrid (MG) methods are iterative methods designed to solve equation systems associated with discretized differential equations and are tailored to the problem to be extremely efficient. Moreover, they are well-suited for Newton-Krylov acceleration. We construct MG methods for approximations to the specific FV discretization which is equivalent to our DGSEM-GL discretization. This allows us to design new and efficient preconditioned DG methods. In the set up of the MG preconditioner, the smoother plays an important role. We consider the pseudo time iteration W3 smoother from [1] with an SGS type approximation of the Jacobian. Numerical results demonstrate the potential of the new preconditioner where we use the FV discretization as optimal reference preconditioner. We show numerical results for the one-dimensional Euler equations.

References

[1] P. Birken, J. Bull, and A. Jameson, *Preconditioned smoothers for the full approximation scheme for the RANS equations*, Journal of Scientific Computing, online first, (2018), https://doi.org/10.1007/s10915-018-0792-9.

[2] T. C. Fisher, M. H. Carpenter, J. Nordström, N. K. Yamaleev, and C. Swanson, *Discretely conservative finite-difference formulations for nonlinear conservation laws in split form: Theory and boundary conditions*, Journal of Computational Physics, 234 (2013), pp. 353–375.

[3] G. J. Gassner, *A skew-symmetric discontinuous Galerking spectral element discretization and its relation to SBP-SAT finite difference methods*, Journal of Scientific Computing, 35 (2012), pp. A1233–A1253.

[4] L. M. Versbach, P. Birken, and G. J. Gassner, *Finite volume based multigrid preconditioners for discontinuous Galerkin methods*, PAMM, 18 (2018), https://doi.org/10.1002/pamm.201800203.