Advances in High-order Residual Distribution Scheme for Fluid Dynamics and Lagrangian Hydrodynamics

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ABSTRACT

In this talk, we will focus on the recent progress in the development of Residual Distribution (RD) schemes for complex flows. We will first discuss the design of a high order finite element type residual distribution scheme in the framework of multidimensional compressible Euler equations of gas dynamics. The strengths of the proposed approximation rely on the generic spatial discretization of the model equations using a continuous finite element type approximation technique, while avoiding the solution of a large linear system with a sparse mass matrix which would come along with any standard ODE solver in a classical finite element approach to advance the solution in time. In this work, we propose a new Residual Distribution (RD) scheme, which provides an explicit and arbitrary high order approximation of the smooth solutions of the Euler equations both in space and time. The design of the scheme allows for an efficient diagonalization of the mass matrix without any loss of accuracy. This is achieved by coupling the RD formulation [1] with a Deferred Correction (DeC) type method [2] for the discretization in time and choosing Bernstein polynomials as shape functions. The advantage of such a matrix-free approach consists in preserving a compact approximation stencil even at high orders, which reduces the computational cost compared to classical finite element techniques and provides potential benefit for exascale computing on future computer architectures. We have assessed our method on several challenging benchmark problems for one- and two- dimensional Euler equations and the scheme has proven to be robust and to achieve the theoretically predicted high order of accuracy on smooth solutions [3].

Robust stabilization (or limiting) techniques are required for high order numerical methods to prevent the generation of non-physical oscillations around the discontinuities. This is especially crucial for the simulations of very strong shock waves. We therefore incorporate and adapt the recently proposed a-posteriori limiting ideas [4, 5] for the RD method in a moving reference frame, which allows for targeted limiting only in the mesh elements exhibiting non-physical behavior and thus help to avoid excessive dissipation of the numerical solution and maintain high accuracy in smooth flow regions. Our numerical results demonstrate the increased accuracy of the RD-MOOD method.

We also show that the proposed approach extends naturally to non-conservative fluid flow models. The pressure-based and internal energy-based formulations are exploited when dealing with nonlinear equations of state and make it possible to obtain oscillation-free solutions, contrary to classical conservative methods. We will present the high-order multidimensional Staggered Grid Residual Distribution (SGH RD) scheme for Lagrangian hydrodynamics. In Lagrangian formulation, the equations are formulated with respect to the moving reference frame and therefore the mesh moves with the flow. The SGH RD scheme is based on the staggered finite element discretization which uses continuous polynomial approximation of the kinematic variables. It can be shown that for the Lagrangian formulation written in non-conservative form, our residual distribution scheme ensures the exact conservation of the total

energy. We shall also discuss stabilization techniques for the SGH RD schemes allowing to reduce the dissipation of the numerical solution. In particular, we show how the excessive artificial viscosity affects the resolution of vorticity and propose using MARS artificial viscosity [6] to improve the accuracy (see Fig. 1).

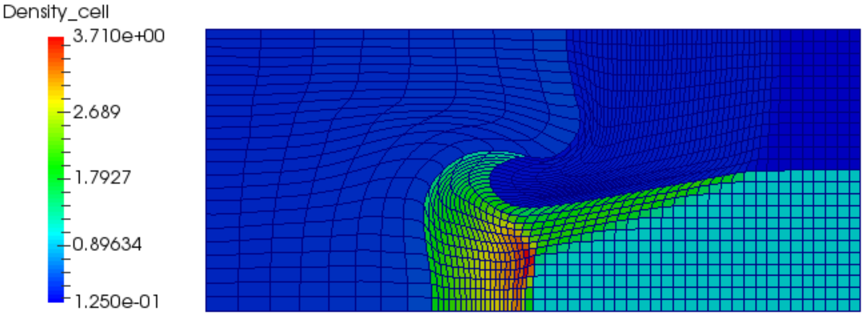

Figure 1: Triple point problem: density computation using MARS viscosity.

Thanks to the generic formulation of the staggered grid residual distribution scheme, it can be directly applied to both single- and multimaterial and multiphase models. Finally, we shall demonstrate computational results obtained with the proposed residual distribution scheme for several challenging test problems involving strong shocks and vortical flows.

References

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