High order numerical schemes for linear and non-linear elasticity

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In this work we solve the linear elasticity equations in complex geometries using two different approaches. In the first part we present a new high order accurate space-time discontinuous Galerkin scheme for the solution of the linear elastic wave equations on staggered unstructured meshes. The method is arbitrary high order accurate in both space and time. Within the staggered mesh formulation, we define the discrete velocity field in the control volumes of a primary mesh, while the discrete stress tensor is defined on a face-based staggered dual mesh. The space-time DG formulation leads to an implicit scheme that requires the solution of a linear system for the unknown degrees of the velocity field at the new time level. Thanks to the use of a spatially staggered mesh, the stencil of the final velocity system involves only the element and its direct neighbors and the linear system can be efficiently solved via matrix-free iterative methods. In the second part, we address the problem of geometrically complex free surface boundary conditions with a novel diffuse interface method on adaptive Cartesian meshes that consists in the introduction of a characteristic function $0 \leq \alpha \leq 1$ which identifies the location of the solid medium and the surrounding air and thus implicitly defines the location of the free surface boundary. Our new approach completely avoids the problem of mesh generation, since all that is needed for the definition of the complex surface topography is to set a scalar color function to unity inside the regions covered by the solid and to zero outside. In order to reduce numerical dissipation, we use high order DG finite element schemes on adaptive AMR grids together with a high resolution shock capturing sub-cell finite volume limiter in the diffuse interface region. Finally, we will move to the Godunov, Peshkov and Romensky model (GPR) in order to describe complete non-linear elasto-plastic phenomena such as dynamic rupture. Also in this case we introduce a damage parameter $0 \leq \xi \leq 1$ so that the fracture propagation is not necessarily aligned with the mesh. The combination of this model with the diffuse interface approach allows to represents geometrically complex free surface boundary conditions and fault lines.

References

[1] M. Tavelli and M. Dumbser *Arbitrary high order accurate space-time discontinuous Galerkin finite element schemes on staggered unstructured meshes for linear elasticity*, Journal of Computational Physics, 366, pp 386-414, 2018.

[2] M. Tavelli and M. Dumbser and D. E. Charrier and L. Rannabauer and T. Weinzierl and M. Bader, *A simple diffuse interface approach on adaptive Cartesian grids for the linear elastic wave equations with complex topography*, Journal of Computational Physics, submitted to, http://arxiv.org/abs/1804.09491.