An Approach to Unsteady p-Adaptation Based on Truncation Error Estimations for High-Order Discontinuous Galerkin Methods

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Abstract. *We present a dynamic p-adaptation algorithm for unsteady problems that targets the truncation error for p-anisotropic Discontinuous Galerkin (DG) discretizations. In order to achieve that, we introduce a methodology to estimate the truncation error of unsteady cases. The proposed method is tested for compressible flow problems, where it provides significant reductions of computational resources (both in storage and CPU-time) as compared to uniform refinement techniques.*

Truncation error estimators based on the τ*-estimation method have been successfully used to perform locally adaptive simulations for steady boundary value problems [1,3,4]. However, there is a lack of scientific literature on their use for unsteady cases.*

Parting from a general partial differential equation,

$$\partial_t \mathbf{q} = \mathcal{R}(\mathbf{q}), \quad (1)$$

the truncation error of a DG discretization of order N ($\partial_t \mathbf{q}^N = \mathcal{R}^N(\mathbf{q}^N)$*) is defined as*

$$\tau^N = \mathcal{R}^N(\mathbf{I}^N \mathbf{q}) - \mathcal{R}(\mathbf{q}). \quad (2)$$

Since the exact solution is generally not known, it is approximated as $\mathbf{q} \approx \mathbf{q}^P$ *(*$P > N$*), and the exact partial differentiation operator is approximated as* $\mathcal{R}(\mathbf{q}) \approx \mathcal{R}^P(\mathbf{q}^P)$*. Therefore, the truncation error estimation yields*

$$\tau_P^N = \mathcal{R}^N(\mathbf{I}_P^N \mathbf{q}^P) - \mathbf{I}_P^N \mathcal{R}^P(\mathbf{q}^P). \quad (3)$$

To compute equation (3), the cheap-to-evaluate decoupled truncation error estimator for tensor-product DG methods derived in [2] is adapted to unsteady problems and the p-adaptation methodology with error-extrapolation that uses an anisotropic 3V multigrid cycle described in [3] is employed as the error estimator.

We analyze the convection of an inviscid compressible vortex (Fig. 1) and the compressible vortex shedding past a cylinder at Re=100 (Fig. 2). A thorough study of the accuracy and computational cost of the method is presented when different truncation error thresholds and adaptation intervals are used, as well as a comparison with uniform refinement techniques and details on how to efficiently implement the proposed methodology. In general, significant reductions in the number of degrees of freedom (DOFs) and computational times are observed for the same levels of accuracy (dispersion and dissipation errors), as compared to uniform refinement techniques (Fig. 3).

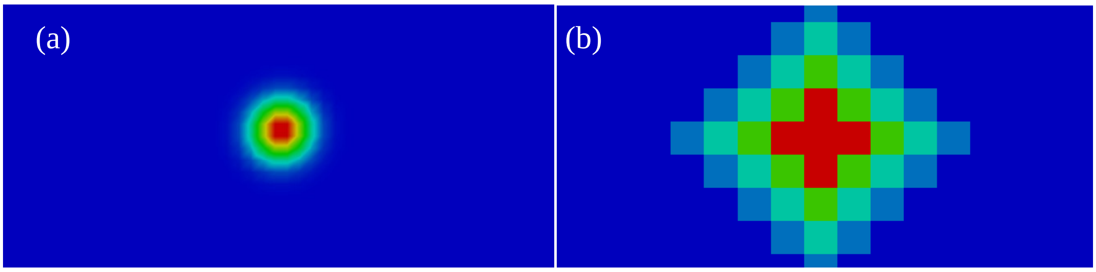

Figure 1. *Contours for density (a) and average polynomial orders (b) for the vortex convection case.*

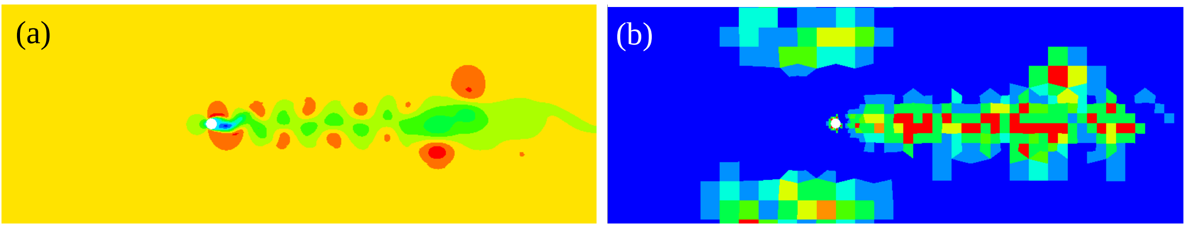

Figure 2. *Contours for x-Velocity (a) and average polynomial orders (b) for the vortex shedding past a cylinder. Re=100.*

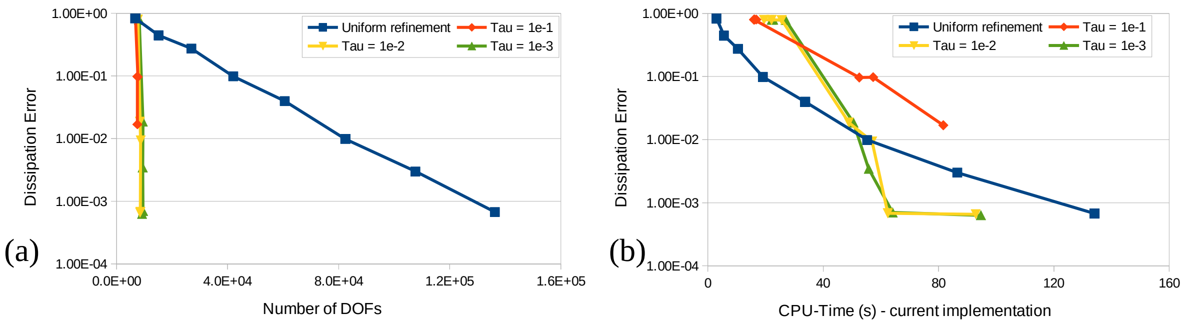

Figure 3. *Obtained dissipation error for uniform refinement and the proposed p-adaptation method vs. number of degrees of freedom (a) and CPU-time (b).*

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