A high-order discontinuous Galerkin solver for multiphase flows

Juan Manzanero[1a], Carlos Redondo[1], Gonzalo Rubio[1b], Esteban Ferrer[1], Eusebio Valero[1], Susana Gómez-Álvarez[2] and Ángel Rivero[2]

[1] ETSIAE-UPM - School of Aeronautics,
Plaza Cardenal Cisneros 3, E-28040 Madrid, Spain
e-mail [a]: juan.manzanero@upm.es e-mail [b]: g.rubio@upm.es
[2] Repsol Technology Lab,
Agustín de Betancourt S/N, E-28935 Móstoles, Madrid, Spain

Abstract. *We present a multiphase model for two-phase incompressible flows. Several approximations can be found in the literature to model the evolution of multiphase flows. Volume of Fluid (VOF) model* (Hirt and Nichols, 1981) *is among the simplest and most popular models. VOF uses one set of momentum equations (shared by the two phases) and defines the concept of volume fraction, which corresponds to the volume occupied by each phase in each infinitesimal control volume. In the VOF model, the evolution of the volume fraction is tracked down by means of an advection equation. In this work, we use a Phase-field method* (Jacqmin, 1999) *that conserves the simplicity of the VOF while providing physical meaning to the evolution equation for the spatial distribution of the two fluids. Therefore, the volume fraction is substituted by a phase-field parameter and the Cahn-Hilliard equation* (Cahn and Hilliard, 1958) *is used to model its evolution. As far as the momentum equation is concerned, our model uses one shared set of the incompressible Navier-Stokes equations for the two-phase flow.*

Both the Cahn-Hilliard equation and the incompressible Navier-Stokes equations are discretised in space by means of a high-order discontinuous Galerkin method. The discontinuous Galerkin method is a popular discretisation method for conservation laws, such as the Navier-Stokes equations (Ferrer, 2017; Fraysse et al., 2016; Manzanero et al., 2018a; Wang et al., 2013)*, essentially due to its geometrical flexibility and computational efficiency. Specifically, we use a nodal variant of the DG method: the Discontinuous Galerkin Spectral Element Method (DGSEM)* (Black, 1999; Kopriva, 2009). *Recently, a method to enhance the robustness of the DGSEM based on split forms has been introduced* (Gassner et al., 2017; Manzanero et al., 2018b) *which turns it into a very interesting choice among the different variants of DG method.*
As far as time integration is concerned, the Cahn-Hilliard equation includes a fourth-order derivative which greatly limits the efficiency of explicit time integration schemes. Therefore, time integration of the system is performed by means of an implicit-explicit method that permits using the same time steps of a typical Navier-Stokes explicit solver while maintaining a computational (per time-iteration) cost similar to an explicit method. For a detailed description of the model, see (Manzanero et al., 2019).

Several validation and test cases, including a spinodal decomposition, a rising bubble and several channel flows are presented to show the validity and the computational efficiency of our approach.

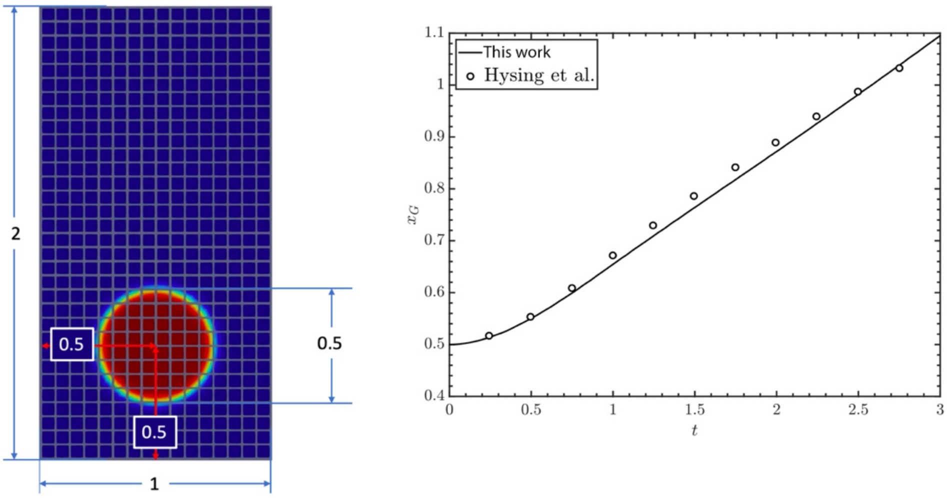

Figure 1 - LEFT. Initial condition and domain for a rising bubble test case. RIGHT. Evolution of the center of mass of the bubble with time compared with a sharp-interface model [11].

REFERENCES

Black K (1999) A conservative spectral element method for the approximation of compressible fluid flow. Kybernetika 35:[133]–146

Cahn JW, Hilliard JE (1958) Free energy of a nonuniform system. I. Interfacial free energy. The Journal of chemical physics 28:258–267

Ferrer E (2017) An interior penalty stabilised incompressible discontinuous Galerkin–Fourier solver for implicit large eddy simulations. Journal of Computational Physics 348:754–775

Fraysse F, Redondo C, Rubio G, Valero E (2016) Upwind methods for the Baer–Nunziato equations and higher-order reconstruction using artificial viscosity. Journal of Computational Physics 326:805–827

Gassner GJ, Winters AR, Hindenlang FJ, Kopriva DA (2017) The BR1 scheme is stable for the compressible Navier–Stokes equations. Journal of Scientific Computing 1–47

Hirt CW, Nichols BD (1981) Volume of fluid (VOF) method for the dynamics of free boundaries. Journal of computational physics 39:201–225

Jacqmin D (1999) Calculation of two-phase Navier–Stokes flows using phase-field modeling. Journal of Computational Physics 155:96–127

Kopriva DA (2009) Implementing spectral methods for partial differential equations: Algorithms for scientists and engineers. Springer Science & Business Media,

Manzanero J, Ferrer E, Rubio G, Valero E (2018a) On the role of numerical dissipation in stabilising under-resolved turbulent simulations using discontinuous Galerkin methods. arXiv preprint arXiv:180510519

Manzanero J, Redondo, Carlos, Rubio G, Ferrer E, Valero E, Gómez-Álvarez, Susana, Rivero, Ángel (2019) A high–order discontinuous Galerkin solver for multiphase flows. (Submitted to) ICOSAHOM 2018

Manzanero J, Rubio G, Ferrer E, Valero E, Kopriva DA (2018b) Insights on aliasing driven instabilities for advection equations with application to Gauss–Lobatto discontinuous Galerkin methods. Journal of Scientific Computing 75:1262–1281

Wang ZJ, Fidkowski K, Abgrall R, Bassi F, Caraeni D, Cary A, Deconinck H, Hartmann R, Hillewaert K, Huynh HT (2013) High-order CFD methods: current status and perspective. International Journal for Numerical Methods in Fluids 72:811–845