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An entropy stable high order DGSEM for the Baer-Nunziato model

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Abstract

In this work we extend the general framework from [1] for the design of high order schemes for nonlinear non-conservative hyperbolic systems, to construct a high order discretization of the Baer-Nunziato model. The space discretization is performed using a discontinuous Galerkin spectral element method (DGSEM) based on collocation between interpolation and quadrature points defined from Gauss-Lobatto quadrature rules. The typical form of the DG scheme allows us to use summation-by-parts operator [3] through which we replace the integration of physical fluxes over discrete elements by entropy conservative fluctuation fluxes similar to Castro et al. [2]. Furthermore, to capture physically relevant solutions across shock discontinuities, we design entropy stable fluxes at element interfaces. The use of entropy stable fluxes allows us to satisfy a semi-discrete entropy inequality across discontinuities.

For the time discretization we rely on strong-stability preserving Runge-Kutta schemes, and under explicit conditions on the time-step and numerical parameters, we prove positivity of the phase densities and a min-max principle on the void fraction for the semi-discrete scheme.

Numerical experiments in one space dimension support the conclusions of the present analysis and highlight stability and robustness of the present schemes. We also plan to extend the scheme to three space dimensions.

References

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