Symmetrizable first order formulation of Navier-Stokes equations and numerical results with the discontinuous Galerkin method

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ABSTRACT

In [1], it was remarked that the heat equation could be formulated as the 0-relaxation limit of a hyperbolic system with source term. This formulation is attractive for the following reasons: it gives a unified discretization of hyperbolic and diffusive terms; it may allow to get a high order representation of gradients [2]; it may allow to relax the stiffness of the advection-diffusion coupling [2]; it may allow to derive non reflecting boundary conditions [3].

The aim of this talk is first to propose an extension of Cattaneos formulation to general dissipative systems such as the compressible Navier-Stokes equations, and prove that the proposed first-order formulation is hyperbolic. The proposed first-order formulation is an alternative hyperbolic system approach to the continuum-based hyperbolic first-order system approach introduced in [4].for the compressible Navier-Stokes equations, and represents, for the first time, a significant improvement to the non-symmetrizable first-order system approach given in [5].

Then a discontinuous Galerkin discretization of the system is proposed, and the benefits of this method in terms of stiffness of the system and accurate representation of the fluxes compared to the classical Navier-Stokes approach are discussed, see Figure 1.

References

[1] Carlo Cattaneo. A form of heat conduction equation which eliminates the paradox of instantaneous propagation. *Compte Rendus*, 247(4):431–433, 1958.

[2] Hiroaki Nishikawa. New-generation hyperbolic Navier-Stokes schemes: O (1/h) speed-up and accurate viscous/heat fluxes. In *20th AIAA Computational Fluid Dynamics Conference*, page 3043, 2011.

[3] Sofia Eriksson and Jan Nordström. Exact non-reflecting boundary conditions revisited: well-posedness and stability. *Foundations of Computational Mathematics*, pages 1–30, 2016.

[4] Ilya Peshkov and Evgeniy Romenski. A hyperbolic model for viscous newtonian flows. *Continuum Mechanics and Thermodynamics*, 28(1-2):85–104, 2016.

[5] Hiroaki Nishikawa. Alternative formulations for first-, second-, and third-order hyperbolic Navier-Stokes schemes. In *22nd AIAA Computational Fluid Dynamics Conference*, page 2451, 2015.

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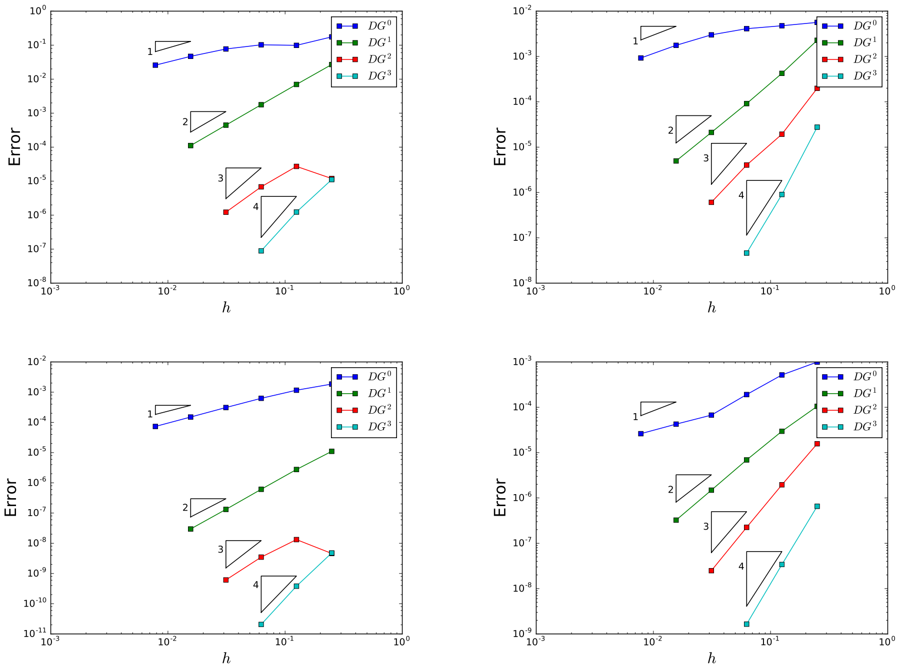

Figure 1: Order obtained on the velocity (top left), temperature (top right), on $\boldsymbol{\tau}_{xy}$ (bottom left) and on $\boldsymbol{q}_y$ (bottom right) for a compressible Poiseuille flow. The observed order matches with the optimal order for the original variables and also for the fluxes.