Analysis of SAT-Techniques in the Finite-Element-Framework

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Simultaneous approximation terms (SATs) are a standard tool in the finite difference community, together with summation-by-parts (SBP) operators, for proving stability results. The main idea about SATs is that the boundary condition is only fulfilled in a weak sense, and also a boundary operator is constructed and applied which guarantees that a discrete energy estimation is simultaneously fulfilled [3].
By introducing SBP operators in the discontinuous Galerkin framework [2], also the SATs technique has been transferred and utilized to prove (entropy)-stability. However, in continuous Galerkin schemes the SATs are -up to our knowledge- not really in use and we want to change this.
In this talk, we focus on this issue. We shortly repeat the main aspects of the SAT technique and demonstrate an approach to construct these boundary operators. Different from the classical ansatz, we switch to entropy variables for the estimation of our operators and consider non-linear problems. In [1] an entropy corrections were introduced to guarantee entropy conservation or even entropy stability for considered schemes. This ansatz is used in a pure continuous Galerkin scheme. Together with the constructed boundary operator, we demonstrate in numerical experiments that the obtained Galerkin scheme is stable through our boundary procedure.

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[2] Gregor J Gassner, *A skew-symmetric discontinuous Galerkin spectral element discretization and its relation to SBP-SAT finite difference methods*, SIAM Journal on Scientific Computing **35** (2013), no. 3, A1233–A1253.

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[3] Jan Nordström, *A roadmap to well posed and stable problems in computational physics*, Journal of Scientific Computing **71** (2017), no. 1, 365–385.