Implicit-explicit schemes for degenerate diffusion-convection PDE

Pep Mulet
Universitat de Valencia, Spain

Implicit-Explicit (IMEX) Runge-Kutta (RK) schemes for partial differential equations with convection and (possibly strongly) degenerate diffusion, that use an explicit RK scheme for the time integration of the convective part and a diagonally implicit one for the diffusive part, are suitable for their much more favourable stability restrictions with respect to explicit integrators and for not having to deal, as fully implicit solvers do, with the fairly sophisticated discretization of the convective terms when they are dominant.

In the scheme of this type that is proposed [Brger, Mulet, Villada, SISC, 2013], the nonlinear and nonsmooth systems of algebraic equations arising in the implicit treatment of the degenerate diffusive part are solved by smoothing of the diffusion coefficients combined with a damped Newton-Raphson method with a line search strategy for globalizing convergence.

To overcome the CPU and implementations costs of these schemes while keeping the advantageous stability properties of IMEX-RK methods, a second variant of these methods is proposed in [Boscarino et al., SISC, 2015]. Now the diffusion terms are discretized in a way that more carefully distinguishes between stiff and nonstiff dependence, such that in each Runge-Kutta stage only a linear system needs to be solved, while still maintaining high order accuracy in time.

These schemes may be advantageous in some cases, but are not advisable in others where special structure of the diffusive terms would be lost. This is the case of some nonlinear convection-diffusion equations with nonlocal flux and possibly degenerate diffusion that arise in many scientific contexts [Carrillo, Chertock, Huang, CCP, 2015], [Brger, Inzunza, Mulet, NMPDE, 2019].

In this talk a survey of these techniques will be given, some recent successful applications of them will be reported and some future applications, as multispecies nonlinear nonlocal equations with cross-diffusion or Navier-Stokes-Cahn-Hilliard equations, will be presented.