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DENSITY POSITIVITY AND MASS CONSERVATION FOR AN IMPLICIT HIGH-ORDER DISCONTINUOUS GALERKIN METHOD APPLIED TO VARIABLE DENSITY INCOMPRESSIBLE FLOWS

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In this work we discretize the set of Variable Density Incompressible (VDI) equations by means of a high-order Discontinuous Galerkin (DG) method. The inviscid interface flux is based on the solution of local Riemann problems perturbed by means of an artificial compressibility term [2]. The viscous interface flux is defined following the BR2 scheme [1]. The resulting system of DAE is integrated in time by means of high-order implicit schemes which allow to exploit the high-order discretization both in space and time and avoid CFL restrictions.

The VDI flow model entails the solution of the system comprising the incompressibility constraint, the momentum and the mass conservation equations. In this model the density behaves essentially as an advected property which can be used as a color function to distinguish different fluids with different densities when dealing with multicomponent problems.

One of the main issues in applying high-order DG methods to such class of flow problems is the treatment of interfaces among different fluids. The approach proposed in this work captures interfaces in a diffuse fashion by high-degree polinomial approximations. In order to control resulting spurious oscillations we add a local artificial viscosity term to the discretization [3, 4].

However, despite the artificial viscosity, oscillations could yet lead to negative values of density. To overcome the problem we propose to use a different working variable for the continuity equation. In particular, the definition of the new variable is based on a hyperbolic function which allows to bound not only the minimum value of density but also its maximum. This change of variable modifies only the partial derivative in time of the DAE system, leaving untouched the right hand side. As a consequence, the DG spatial discretization still ensures the conservation of mass but the time integration introduces an error which is proportional to the global error of the temporal scheme.

Results on numerical experiments involving also high-density ratios have been obtained using very high-order polynomial approximations on relatively coarse grids.

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