A class of polytopial $H(\mathrm{div})$ – conformal elements and their approximation spaces

Rémi Abgrall, Élise Le Mélédo and Philipp Öffner

January 15, 2019

Abstract

The Raviart – Thomas elements are widely used when defining numerical scheme (*e.g.* (mixed) finite volumes, (mixed) finite element methods, flux reconstruction) due to the interesting properties of conservation they have. However, their classical definition is limited to quads and simplicial elements, which prevents the use of general polytopial meshes.

We propose here a new type of approximation spaces, bridging the flexibility of the Virtual Element space [1] and the conformity - friendly property of the Raviart – Thomas's one [2]. Depending on the polytope's shape, it allows the construction of $H(\mathrm{div})$ – conformal elements for any discretisation order.

A class of $H(\mathrm{div})$ – conformal elements is then constructed through the definition of degrees of freedom, that can be categorised into four different subtypes. Within each, their possible definition is flexible and allows an element's construction on any polytopial shape (possibly up to rotation or translation). Furthermore, by construction any element has the same properties than the Raviart – Thomas ones at the cells interfaces, for any order.

The two dimensional case is studied in details and a practical construction of basis functions for one specific element is shown as an example. This work can be applied to construct numerical methods on polytopes (*e.g.* in the flux reconstruction context [3]).

References

[1] L. Beirão Da Veiga, F. Brezzi, A. Cangiani, G. Manzini, L. D. Marini, and A. Russo. Basic principles of virtual element methods. *Mathematical Models and Methods in Applied Sciences*, 23(01):199–214, jan 2013.

[2] P. A. Raviart and J. M. Thomas. A mixed finite element method for 2-nd order elliptic problems. In Ilio Galligani and Enrico Magenes, editors, *Mathematical Aspects of Finite Element Methods*, pages 292–315, Berlin, Heidelberg, 1977. Springer Berlin Heidelberg.

[3] Rémi Abgrall, Élise Le Mélédo, and Philipp Öffner. On the connection between residual distribution schemes and flux reconstruction. *arXiv preprint arXiv:1807.01261*, 2018.