A Field-Guided Method for Quadrilateral Mesh Generation: Using High Order Methods to Generate Grids for High Order Methods

David Kopriva, Julian Marcon, Joaquim Peiro, Spencer Sherwin

March 3, 2019

Despite the advantages of tensor product spectral element approximations on quadrilateral meshes, e.g. efficient computation of derivatives and high aspect ratios allowed for boundary layer computations, the methods' applicability has been limited by the difficulty in generating quadrilateral meshes. In recent years, ideas from the computer science community regarding surface representations, most notably cross-field methods, have been proposed to be used for mesh generation. These methods generate quadrilateral decompositions of a domain with a minimum number of irregular points, i.e., interior points associated with fewer or more than four elements, making the grids thus generated suitable for finite element, block structured finite difference, or spectral element methods. The approaches rely on low order techniques to generate a cross-field on a triangular mesh. The cross-field is then followed from singular points to generate streamlines – curves that form the edges of the quadrilateral subdomains. The use of low order techniques often lead to the detection of extraneous irregular points, streamlines that form limit cycles, and require ad hoc treatment of corner points along boundaries that contain angles that are not a multiple of pi/2.

We will describe a high order technique to generate quadrilateral decompositions and meshes for complex two-dimensional domains using spectral elements in a field guided procedure. Inspired by cross-field methods, we never actually compute crosses. Instead, we compute a high order accurate guiding field using a continuous Galerkin (CG) or discontinuous Galerkin (DG) spectral element method to solve a Laplace equation for each of the field variables using the open source code Nektar++. The use of a spectral method provides spectral convergence and sub-element resolution of the fields. The use of a DG approximation allows meshing of corners that are not multiples of pi/2 in a discretization consistent manner, when needed. The high order field can then be exploited to accurately find irregular points, and accurately integrated using a high order streamline integration method to avoid features like limit cycles. The result is a mesh with naturally curved quadrilateral elements that do not need to be curved a-posteriori to eliminate invalid elements. The mesh generation proce-

dure is implemented in the open source grid generation program NekMesh.