A high-order local Discontinuous Galerkin solver for viscoelastic flow: new ways to solve the confined cylinder benchmark problem

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Abstract

In numerical simulations of viscoelastic flow a breakdown in solver convergence often occurs for different computational approaches at critically high values of the Weissenberg number. This is due to problems concerning stability in the discretization and the physical stability of the model. First, we have for steady problems a mixed hyperbolic-elliptic system of equations combined with a convection-dominated convection-diffusion problem in the constitutive part [3]. Second, instabilities occur in the flow field in regions with very steep velocity gradients and high stresses, e.g. in the confined cylinder benchmark problem at the boundary layer of the cylinder and in the wake of it [2]. This leads to a transition to unsteady flow at quite low Weissenberg numbers even for creeping flow (Re $\rightarrow 0$).

We introduce a fully coupled solver for viscoelastic Oldroyd B flow with an exclusively high-order Discontinuous Galerkin (DG) scheme for all equations. The DG method is a promising approach for convection dominated problems and is often used in combination with streamline upwinding for the convective terms of the constitutive equations of viscoelastic flow whereas the other terms are discretized using a Finite Elements ansatz (FEM). Instead, we are using a local DG formulation with penalized fluxes in order to solve the hyperbolic constitutive equations known from the field of DG-discretization of hyperbolic compressible Navier-Stokes equations [1]. The main advantage of our exclusively DG scheme is the fully coupled algorithm omitting the standard methods for viscoelastic flow like the elastic-viscous stress splitting (EVSS) and its derivatives, in which additional evolutionary equations for the velocity gradients are introduced.

To overcome numerical problems caused by physical instabilities which lead to very steep gradients of velocity and stresses with a forming singularity in the wake of the cylinder we use a shock capturing method combined with an artificial viscosity ansatz also known for the numerical solution of compressible Navier-Stokes [4]. This method is used to help the solver finding a close by solution and the viscosity is eliminated step-by-step until the final solution does not contain any artificial diffusion. This has proven necessary since due to the high accuracy of the DG method there was not enough numerical diffusion to let the solver properly converge without it.

The successful implementation of the high-order DG scheme is presented and results are shown for the confined cylinder benchmark problem for steady and unsteady viscoelastic flow.

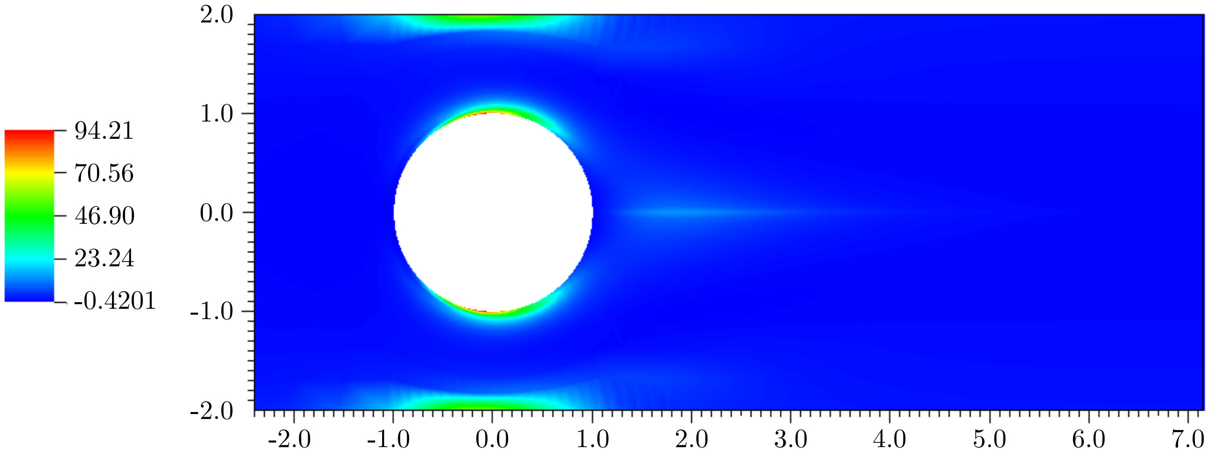

Figure 1: Colour plot of the magnitude of the normal stress component τ_{xx} at $\mathrm{Wi} = 0.6$ in the confined cylinder benchmark problem for steady calculation.

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