**A posteriori sub-cell finite volume limiting of staggered semi-implicit discontinuous Galerkin schemes for the Euler equations of gasdynamic**

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ABSTRACT
We propose a novel semi-implicit Discontinuous Galerkin (DG) scheme on staggered meshes with *a posteriori* sub-cell finite volume limiting for the one and two dimensional Euler equations of compressible gas dynamics. We follow the strategy adopted by Dumbser and Casulli in 2016 (see [1]) where the Euler equations have been solved by using a semi-implicit finite volume method based on the flux-vector splitting proposed by Toro and Vàzquez-Cendòn (see [2]). In particular, the non-linear convective terms are discretized explicitly and then the pressure is discretized implicitly and it is obtained by solving a linear system. As consequence, the time step is given by a mild CFL condition based only on the fluid velocity which makes this new method suitable for simulations in the low Mach number regime.
In addition, in order to deal with shock waves or strong discontinuities, the scheme includes the *a posteriori* sub-cell finite volume limiting technique. This strategy was proposed by Dumbser et al. in 2014 for explicit DG schemes and it is based on the MOOD algorithm of Clain, Loubère and Diot (see [3,4]). Recently, this approach was extended to semi-implicit DG scheme on staggered meshes for the shallow water equations in [5]. In particular, first, at time $t^n$ the unlimited DG scheme produces a so-called candidate solution for the time level $t^{n+1}$. Later on, the control volumes with a non-admissible candidate solution are identified by using physical and numerical detection criteria in order to check the positivity of the solution, the absence of floating point errors and the respect of a relaxed discrete maximum principle (DMP). Then a robust, stable in the sense of Godunov, first order semi-implicit finite volume (FV) method is applied on a sub-grid composed of 2P + 1 cells where P denotes the polynomial degree used in the DG scheme. Successively, the nonlinear convective terms are update by using the well known a posteriori subcell limiter for explicit DG scheme while the linear system for the new pressure is assembled and solved again. Finally, the higher order DG polynomial are reconstructed from the piecewise constant subcell finite volume averages and the scheme proceeds with the next time step.
We validate this novel family of methods and we carry out the classical numerical benchmarks of gas dynamics. Great attention is dedicated to 1D and 2D Riemann problems and we show that for these test cases the scheme works well in the presence of shock waves and it does not produce non-physical oscillations.

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