A Discontinuous Galerkin immersed boundary solver for compressible flow: From time efficient shock-capturing to shock-fitting

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Abstract

In this talk we study unsteady high Mach number flows in the context of the Euler equations. We use a Discontinuous Galerkin (DG) solver [Müller et al., 2016] that was extended to immersed boundaries where the considered geometry is represented by the zero iso-contour of a level set function. We apply cell-agglomeration that averts problems with small and ill-shaped cut cells. In order to obtain a stable and accurate solution, discontinuous flow phenomena such as shocks have to be resolved properly, see Figure 1. Particularly, we use a shock-capturing approach based on artificial viscosity. Recently, we coupled both approaches in order to reuse the geometrical flexibility of immersed boundaries.

For shock-capturing we follow the two-step strategy presented by Persson and Peraire [2006]. First, a shock sensor that is based on the modal decay of the DG coefficients detects troubled cells. Second, artificial viscosity is applied to these cells in order to smooth the solution.

To advance the solution in time, we use an explicit time integration scheme. In the aforementioned setting a severe time step restriction is caused by the additional diffusive term that leads to a drastic reduction of computational efficiency. We tackle this problem by using an adaptive local time stepping (LTS) approach [Winters and Kopriva, 2014] that dynamically (re-)partitions the grid into cell clusters after several time steps according to their local time step restrictions. For such applications we have developed a combination with a dynamic load balancing strategy that enables a performant parallelization based on the LTS cell clusters.

Our current work is focused on shock-fitting techniques (see Salas [2010] for an overview) applied to an extended DG (XDG) method [Kummer, 2016] in order to regain the desirable DG convergence rates. There are two main challenges: First, we investigate *shock tracking* in 1D, where we prescribe the flow properties across shocks using the Rankine-Hugoniot conditions in combination with the level set propagation speed. Second, we consider the *formation of stationary shock fronts* in 2D.

Meeting all these challenges, we present results of a new efficient DG immersed boundary solver for compressible flow by combining a shock-capturing scheme with an adaptive LTS method. Moreover, the current status of our new XDG shock-fitting approach will be presented.

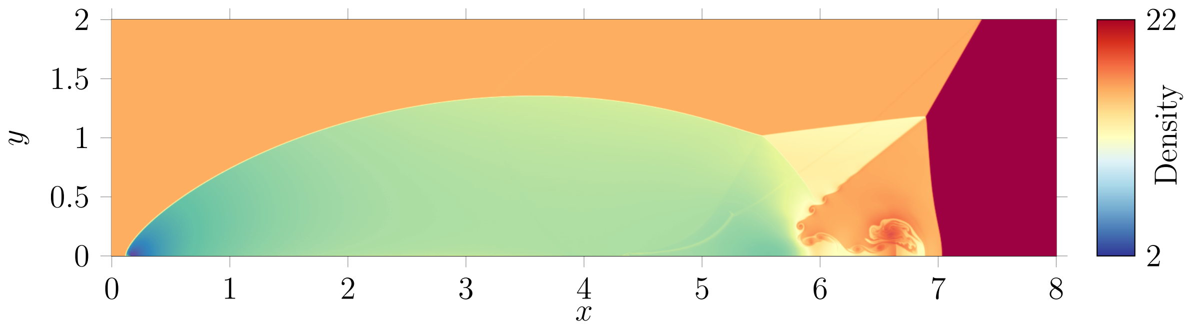

Figure 1: Double Mach reflection with a polynomial order of $P = 3$ on a boundary-fitted grid with 1600×400 cells on the domain $[0, 8] \times [0, 2]$ using an artificial viscosity based shock-capturing technique together with adaptive local time stepping.

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