High order ADER schemes for a first order hyperbolic formulation of compressible flow with surface tension

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In this work, we present a modification to the weakly hyperbolic model for two-phase flow with surface tension forwarded in [1, 8]. The model is based on the one-velocity, one-pressure, two-phase flow model of Kapila *et al.* [6], with surface-tension forces taken into account in a conservative fashion by means of an associated stress tensors. The tracking of the interfaces is achieved by evolving, rather than a scalar tracer, a vector field as in [8], so that the surface-force stress tensor can be expressed directly as a function of the state variables, without the need to compute any gradients of the scalar tracer.

The modification here proposed, motivated by the theory of Symmetric Hyperbolic Thermodynamically Compatible systems [4, 5, 7], is intended to restore the eigenvector that was reported missing in [8], thus obtaining a strongly hyperbolic system of equations.

For the numerical solution of the governing equations, which constitute a hyperbolic system of partial differential equations with non-conservative products, the ADER Discontinuous Galerkin (DG) schemes with a-posteriori finite volume subcell limiting [2, 3, 9] are employed; we carry out a number of numerical tests concerning flows dominated by surface tension and highly compressible flows involving shockwaves and large density ratios, with positive results comparing with either approximate analytical formulas or experiments. Furthermore, by comparing with a new exact analytical solution to the proposed model, we give convergence results for the ADER-DG schemes of order of accuracy up ten.

References

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