High-order Flux Reconstruction schemes with Implicit time-stepping for the Compressible Navier-Stokes equations

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Abstract

High order methods are well suited for Large Eddy Simulations (LES) of turbulent flows as they have low numerical dissipation and high order of accuracy. Such methods based on a discontinuous polynomial representation of the solution like Discontinuous Galerkin (DG) methods [1, 2] are gaining popularity for CFD simulations owing to their locality of data and operations, and ease of h/p adaptation on unstructured grids. In addition to DG schemes which are based on the integral weak form of a conservation law, schemes based on the differential form of the equation have also been developed. Two such schemes are Flux Reconstruction (FR) and Spectral Difference (SD) [3, 4], and their formulations don't depend on explicit numerical quadrature. The FR scheme, introduced by Huynh [5], provides a unifying framework to recover other high-order schemes like the nodal DG [6] and SD method under some specific choice of parameters. The advantage of a DG scheme like data locality is also shared by the method and its algorithmic simplicity makes it a promising alternative for an industrial CFD solver. However, combining explicit time-stepping with high-order methods is constrained by a restrictive Courant-Friedrichs-Lewy (CFL) condition where a stable time step scales with p^{-2} (Eq. 7.32 [6]), where p is the polynomial order. Moreover, a typical aerodynamic simulation has varied length and time scales associated with the mesh and an explicit time integrator will be restricted by the global CFL constraint. This limitation is purely numerical in nature, and hence motivates the development of an implicit FR formulation to resolve the flow with a time scale of desired accuracy.

In this work, we have considered Explicit Singly Diagonal Implicit Runge-Kutta (ESDIRK) schemes for the implicit formulation where both the inviscid and viscous fluxes are evaluated implicitly. For viscous flow problems, time step restrictions are much more severe for the parabolic terms and a stable time step scales as h^2 instead of h with h being the mesh size. Based on the premise that the acoustic time scales should be resolved for unsteady simulations, we have also considered Implicit-Explicit (IMEX) Runge-Kutta schemes [7] to treat the viscous flux implicitly while integrating the inviscid flux explicitly in the inviscid CFL limit. In FR discretization, the Rusanov flux is used to compute the inviscid numerical flux at an inter-element interface and the differential form of Local Discontinuous Galerkin (LDG) [8] is applied for computing the viscous flux. The LDG flux is not compact on a general 2D grid, resulting in high memory storage for the Jacobian matrix and increased iterations for the linear solver. To alleviate these issues, viscous fluxes based on compact discretizations, for example Compact Discontinuous Galerkin (CDG) [9] and the second Bassi-Rebay (BR2) [10], are considered. Using a compact flux led to a significant improvement in the efficiency of the scheme.

The implicit FR scheme requires the solution of nonlinear equations obtained in every stage of the time integrator. The Newton-Raphson method is used for solving the nonlinear equations in a matrix-based approach where the Jacobian of the residuals is computed exactly and stored in a sparse block matrix format. The exact computation can be efficiently performed as the operations are mostly element-local, and offers the possibility of creating an effective preconditioner for a Krylov subspace method. The GM-RES method is used for solving the large sparse linear system with the Jacobi preconditioner based on the diagonal blocks of the matrix. The accuracy and performance of both semi-implicit and full-implicit FR will be presented on 2D canonical test cases like 2D flow over a cylinder and over a SD7003 airfoil at 8^o angle of attack. Numerical results have demonstrated that in high Reynolds number flows, the implicit FR based on the ESDIRK scheme is more efficient ($\sim 50\%$) than the explicit one. In contrast,

the semi-implicit FR based on the IMEX scheme is efficient only in the low Reynolds number regime where viscous effects are dominant. Although our expectation for the efficiency of the semi-implicit FR was not realized for a typical aerodynamic flow at a high Reynolds number due to low viscosity, it can still be used for integrating stiff terms usually found in the transport equations of RANS models [11]. Our future work consists of extending the implicit FR formulation to 3D problems and assessing its performance for benchmark test cases. To improve its efficiency, convergence acceleration techniques like h/p multigrid will also be investigated.

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